How to apply the Fishbone diagram in a cutting tool manufacturing company setting

Training Material by Astakhov Tool Service

Module 1: Methodology

Introduction

There is an increasing amount of pressure on cutting tool manufactures to meet stringent quality and delivery specifications at lower process. Strategic process improvements are needed to increase profit margins while meeting the dynamic needs of the customers. However, tool manufactures often possess a firefighting mentality, becoming overwhelmed with day-today problems. Often, these problems are common cause, chronic issues where tool manufacturers losing sight of what needs to be done to make systemic process changes having long lasting consequences.

Due to excessive firefighting, tool manufactures often incur significant cost of poor quality of their tools resulting in ‘unexpected’ yet repeated tool failures. Unfortunately these failures are not properly documented and, as a result, never properly understood. Many filled RMA forms in the section “Root cause of tool failure” contain simple recognition of the fact: “Tool broke.” As a result, a way too many CT manufacturers do not have a proper tool failure database and therefore a system in place to correct many systemic problems. There is nobody directly responsible for analyzing these failures because CT manufactures often do not have qualified people to do so.

Six Sigma is a strategic business improvement approach that increases both customer satisfaction and a CT manufacturer financial health through systemic process change. However, it can be a great success or an expensive failure, depending upon how it is deployed. To assure successful implementation of Six Sigma (as well as lean manufacturing), the following conditions should be satisfied:

- It starts from the top management that reinforces this process on the daily basis.
- It is ongoing process of infusing the Six Sigma methodology into CT manufacture culture.
- It is viewed by all company employees as the top priority that assures company’s bright future.
Rule #1: If the top management of a CT manufacturing company decides to implement the Six Sigma approach, at least a vice-president of the company should be responsible for this implementation. Moreover, he/she should lead all activities in this direction, should reinforce Six Sigma procedures and policies, and should first pass Six Sigma training remaining the most informed person on the matter all the time.

The Six Sigma business approach should not be confused with lean manufacturing known also as the Toyota Production System. The former is total improvement of a CT manufacturer process (concept, design, manufacturing, implementation, customer support etc.) while the latter concerns only improvements in manufacturing. It a CT company does not have a decent product in the first place, the implementation of lean manufacturing just helps produce the existing junk product more efficiently and deliver it JIT.

One of the most important yet less understood pillar of Six Sigma is the Fishbone diagram. It is one of the simplest problem-analysis tools that can be used to analyze cutting tool (hereafter CT) relate issues. This training material introduces you to the Fishbone diagram and as an example, applies it to an actual CT issue.

In any organization problem analysis and management tools are crucial to success. In cutting tool quality management, there are a number of different formal and informal tools that can be used. Two of them, the Fishbone diagram and the Pareto principle, are most common and easy to implement. In this training material, you are to be introduced to the problem analysis tool known as the Fishbone diagram and to the management principle known as the Pareto principle. A discussion on how these techniques are relevant to CT business and how to use them will be carried out using simple examples. The purpose of the Fishbone diagram is not to find solutions to CT-related problems, but to determine the root cause of a problem. It may also be considered as a graphical representation of “5W=1H” root cause analysis used in lean manufacturing.

This training material is intended for experienced CT designers, manufactures and user as well as for managers with little or no knowledge of the Fishbone diagram.

Fishbone diagram: Start with clear formulation of the problem

The Fishbone diagram is also known as the cause and effect diagram, the root cause analysis, and the Ishikawa diagram, named after its originator (first used in 1943) Dr. Kaoru Ishikawa of the University of Tokyo, the Japanese quality pioneer. It is generally called the Fishbone diagram because the diagram resembles that of a fishbone. In simple terms, Fishbone is brainstorming (“5W=1H”) in a structured visual format. This diagram is used to identify all of the contributing root causes likely to be causing a problem. This methodology can be used on any type of problem, and can be tailored by the user to fit the circumstances. Use of this tool has several benefits to process improvement teams:

- Straightforward and easy to learn visual tool.
- Involves the workforce in problem resolution - preparation of the fishbone diagram provides an education to the whole team.
- Organizes discussion to stay focused on the current issues.
- Promotes “System Thinking” through visual linkages.
- Prioritizes further analysis and corrective actions.

The technique uses graphical means to relate the causes of a problem to the problem itself, in other words, to determine cause and effect. The better the problem is
formulated, the better correlation “causes-effect” can be established. To do that, the problem should be easily ‘measurable’ so the correlations “causes-effect” can be established quantitatively and the improvement(s) obtained in the result of the efforts can be clearly seem. Therefore

**Rule #2: Formulate problem in clear terms relevant to CT.** Moreover, the problem formulation should depend upon the technical level of the group which trays to solve that problem. Each group member has to understand the problem in sufficient depth. The better the understanding of the problem, the more efficient is brainstorming. In CT business, the problem must be formulated in CT relevant terminology.

For example, the problem formulated as “Poor tool life” does not formulated in technical terms because the criterion (criteria) of tool life is not specified so the problem is **NOT FORMULATED** in CT terms. As known, there are a number of tool life criteria:

- Tool catastrophic failure or breakage. In this case tool life is measured by the number of parts produced between two successive tool failures or breakages. In the automotive industry, the number of tool failures (breakages) per 1000 part produced often used to average the scatter in tool life measured data.
- The selected width of the flank wear land on the tool flank or the depth of the crater on the rake face (STANDARD ANSI/ASME B94.55M -1985). In this case tool life can be measured in units of time needed to achieve the selected tool life criteria; by the number of parts produced during this time; by the length of the tool path; by the area of the machined surface etc.
- Dimension (sometimes, surface finish) tool life can be characterized by the time within which the tool works without adjustment or replacement. In this case tool life can be measured in units of time; by the number of parts produced during this time; by the length of the tool path passed; by the area of the machined surface etc.

Therefore, a clear and measurable criterion (criteria) should be select is the Fishbone methodology is to be applied.

The diagram focuses on the causes rather than the effect. Because there may be a number of causes for a particular problem, this technique helps one to identify the root cause of the problem in a structured and uncomplicated manner. It also helps one to work on each cause prior to finding the root cause.

This technique is very much applicable to the cutting tool industry and to ends users of cutting tools. When there are problems with cutting tool applications, root cause analysis is important. For example, tool life problems can occur for a number of reasons, including tool design and manufacturing problems, selection of the proper tool for a given application, selection of the proper cutting regime, variation in the properties between different batches of the workpieces (particularly important when these obtained from different suppliers), coolant properties, condition, and regime or other factors. The Fishbone diagram helps the team to arrive at the root cause of a problem through brainstorming.

**Fishbone diagram: Brainstorming**

There are a number of different techniques that can be used for problem solving. They can be broadly divided into two principally different categories: non-algorithmic and algorithmic. Among non-algorithmic methods the following are most known: Brainstorming (its highest, well-organized level sometimes called as Brainattack), Morphological Analyses, Random Output, Provocation, Focal Object etc. Among algorithmic methods, TRIZ is the most popular. The difference between non-algorithmic and algorithmic methods is that the
application of the former DOES NOT guarantee the solution to the problem while the latter normally assures the best possible solution. However, the use of TRIZ requires special training of the group members while non-algorithmic methods can be used with relatively untrained groups.

Brainstorming is the simplest way suitable for the discussed application. Brainstorming is an excellent way of developing many creative solutions to a problem. It works by focusing on a problem, and then coming up with very many radical solutions to it. Ideas should deliberately be as broad and odd as possible, and should be developed as fast as possible. It is designed to help an individual and/or a group to break out of normal thinking patterns into new ways of looking at things.

Group brainstorming can be very effective as it uses the experience and creativity of all members of the group. When individual members reach their limit on an idea, another member’s creativity and experience can take the idea to the next stage. Therefore, group brainstorming tends to develop ideas in more depth than individual brainstorming. It works well however, if there is a well-trained moderator who helps the flow and pick up of ideas, who assures that any idea, even one spelled in very soft vice, is heart by other members of the group, who understands that Brainstorming in a group can be risky for individuals. Valuable but strange suggestions may appear stupid at first sight. Because of such, the moderator needs not to chair but to direct sessions tightly so that uncreative people do not crush these ideas and leave group members feeling humiliated.

To run a group brainstorming session effectively, the following is recommended:

- Define the problem to be solved clearly, and lay out any criteria to be met (see above). Normally, it has to be done before a brainstorming session and discussed with each group member so everyone should be on the same page.
- Pres-select two group of people. The first one is called generators and the second – critics. It is a sole responsibility of a good facilitator to select the members of these two groups properly. Experience shows that this is THE MOST critical yet normalized step in Brainstorming. Where possible, participants in the brainstorming process (both – generators and critics) should come from as wide a range of disciplines as possible. This brings a broad range of experience to the session and helps to make it more creative. What is to be avoided at any cost is “boss-subordinate” relationship in the group members. In other words, group members
- The facilitator has to keep the session focused on the problem.
- The facilitator ensures that no one criticizes or evaluates ideas during the session. Criticism introduces an element of risk for group members when putting forward an idea. This stifles creativity and cripples the free running nature of a good brainstorming session.
- The facilitator encourages an enthusiastic, uncritical attitude among members of the group. Try to get everyone to contribute and develop ideas, including the quietest members of the group.
- The facilitator should let people have fun brainstorming. He/she should encourage them to come up with as many ideas as possible, from solidly practical ones to wildly impractical ones. Any creativity should be most welcomed.
- The facilitator ensures that no train of thought is followed for too long – he/she should notice the point of saturation when no new ideas are coming along. As such, the facilitator should make a smooth transition into the next stage or to a new lead.
- The most important responsibility of the facilitator is to encourage people to develop other people’s ideas, or to use other ideas to create (generate) new ones. We
observed during last Olympic Games realizing the cost of a sloppy baton change during a relay.

- A person should be appointed to note down ideas that come out of the session. A good way of doing this is to use a flip chart. This should be studied and evaluated after the session. Naturally, this person should be equipped with all necessary audio-video means to correct and to add to the notes he/she made during the session.

**Rule #3:** First, find out and develop smart facilitator who then develops the group of generators. Second, once developed, it is better to keep this group with minimum replacements deploying it to solve problems of gradually increased complexity.

The second group (the critics) should not be in the same room with the generators while the brainstorming takes place. Their objective is to provide a critical analysis of the brainstorming results “picking up” useful ideas and filling a Fishbone diagram.

**Fishbone diagram: When to use it**

The Fishbone diagram is used in the following cases:

- To analyze and find the root cause of a complicated problem.
- When there are many possible causes for a problem.
- If the traditional way of approaching the problem (trial and error, trying all possible causes, and so on) is very expensive and/or time consuming.
- The problem is very complicated and the project team cannot identify the root cause.

**Fishbone diagram: Relevance to CT application support**

Most of CT designers and application specialists have some experience in supporting of their CT. You have probably solved problems ranging from the simple to the complex that take anywhere from a few minutes to hours or even days to resolve. For the problems that stumped you most, you may have approached your colleagues, experience practitioners, friends, technical experts (University professors, for example), or others for help. You might even have uncovered a lot of potential causes for a problem without knowing their actual relevance to the problem context, and you might have analyzed each and every one of them. This can lead to increased time taken to find the root cause of the problem.

Using the Fishbone diagram, you can approach the same problem in a more systematic and uncomplicated manner. After listing the possible causes for a problem, you and your team analyze each one carefully, giving due importance to statements made by each team member during the brainstorming session, accepting or ruling out certain causes, and eventually arriving at the root cause of the problem. In general, Fishbone diagrams give you increased understanding of complex problems by visual means of analyses.

**Fishbone diagram: How to construct**

Here are the various tasks involved in constructing a Fishbone diagram:

1. Define and properly formulate the problem
2. Brainstorm.
3. Identify causes.
Define the problem

Although the first step is fairly simple and straightforward, it is of great importance to the whole process. A critical problem needed a permanent fix and that worth brainstorming with the team for which the root cause has to be identified should be formulated in CT terms (see above). The six sigma leader and facilitator should do it together interviewing the responsible manager (engineer). As stated above, the problem should be defined with its clear metric(s).

Rule #4: Proper problem identification and assigning to it the proper metric assures to a large extend the finding of its root cause and its proper fix. Do not hurry up at this stage tons of time and money can be wasted if this stage is not properly completed.

After the problem is identified, the leader and facilitator can start constructing the Fishbone diagram using feedback from the responsible manager (engineer) and other sources of information. Normally, a template similar to that shown in Fig. 1 is used.

![Fishbone Template](image)

Figure 1. Fishbone template.

Brainstorm

People have difficulty understanding how to structure the thought process around a large problem domain. Sometimes it is useful to focus on logically related items of the problem domain and to represent them in the Fishbone diagram, which will convey the problem solving methodology. There are quite a few tools available that can help us in this regard, including:

- **Affinity Chart**
  Organizes facts, opinions, ideas, and issues into a natural grouping. This grouping is in turn used as an aid in diagnosing complex problems.
Brainstorming
Gathers ideas from people who are potential contributors. This process is discussed further in the following sections.

Check sheet
Acts as a simple data recording device that helps to delineate important items and characteristics to direct attention to them and verify that they are evaluated.

Flow charts
Organizes information about a process in a graphical manner and makes it clear who is impacted at every stage.

No single methodology is applicable to all problem domains. Based on experience and study, the six sigma leader has to identify, thoroughly analyze, and maintain the methodology and the related problem domains.

Categorize
When the Fishbone technique is applied to CT problems, the possible causes are usually classified into six categories:

- Methods
- Manpower (People)
- Management
- Measurements
- Materials
- Machines

Though the above are a few important problem categories, during the brainstorming session, the facilitator should encourage the team to come up with all possible categories. The above categories give the team direction to help find the possible causes. Some of the categories listed above may or may not be applicable to a particular case. Let's look briefly at each category (Table 1).

<table>
<thead>
<tr>
<th>Category</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methods</td>
<td>Methods are ways of doing things or the procedures followed to accomplish a task. The established methods should be formalized in the corresponding work instructions for each and every operation used to produce the product, for example a cutting tool. A typical cause under the Method category is a method that does not assure the required quality, the work instructions describing a method are not clear, confusing or simply wrong etc.</td>
</tr>
<tr>
<td>Manpower</td>
<td>People are responsible for the problem because they make products. The problem may have been caused by people who are inexperienced, who cannot answer prompted questions, who do not have sufficient training, who do not follow work instructions and so on.</td>
</tr>
<tr>
<td>Management</td>
<td>Management refers to a wide variety of management strategies and decisions which are not originated from the proper evaluation of customers' perception of quality.</td>
</tr>
<tr>
<td>Measurements</td>
<td>Measurement refers to metrics that are derived from a project. Problems may occur if measurements are wrong or the measurement technique used is not relevant.</td>
</tr>
</tbody>
</table>
| Materials   | Material basically refers to two basically different categories that should be treated in two separate “bones” of the diagram (CTMat and WorkMat, for example). First bone refers to the materials assigned by the tool designer and used in the manufacturing of the cutting tool under consideration (hereafter referred to as the problem cutting tool) – quite often they are not the same because the shop often uses what is available instead what is required to meet the delivery date. Second bone refers to the work material (material to
be cut). Often, the variation of the properties of the work material causes problems in many cutting tool applications. This is particularly true if the cutting tool users (the customers) buy the work materials from different suppliers and/or works with its materials supplier on the metallurgy of the work materials to meet some specific requirement of the end product. As a rule, the cutting tool manufacture is not informed about this activities and thus about multiple alternation of the properties of the work material.

Machines

In CT industry, this category refers to two principally different groups of machines. The first group consists of machines used in CT manufacturing while the second group refer to the machining system(s) that uses the produced cutting tool. Because these two groups may have pronounced but different effect on CT performance, theses two group should be clearly distinguished as MahineProd and MachinAppl groups in the fishbone diagram analysis. One should clearly realize that the first group (MachineProd) affect tool quality itself while the second group causes systemic problems if a particular machining system is considered. There are a lot of possibilities that a problem can be due to the MachinAppl group flaws such as performance issues, incorrect cutting regime, poor coolant quality, alignment problems, insufficient rigidity, wrong process control parameters etc.

Other possible categories include policies, procedure, technology, and so on. More categories are included the better the analysis. As such, however, it becomes more complicated and thus time consuming.

After identifying a problem, the six sigma leader and facilitator gather information about the possible causes through brainstorming, finally arriving at the root cause. The brainstorming team (both the generators and the critics) can either analyze each of the above categories for possible causes or look into other categories (not listed above).

**Identify causes**

While brainstorming, the facilitator should direct the team toward identifying major causes (categories) first, which can be further discussed, and then secondary causes for each major cause can be identified and discussed. This helps the team to concentrate on one major cause at a time and to refine further for possible secondary causes.

After the major categories (usually four to six) are identified, they are placed into a fishbone diagram template as fishbones adding. They are represented as inclined lines with the arrow pointing towards the backbone of the fish.

Sometimes it is difficult to arrive at a few major causes. The team may come up with a lot of causes, which makes brainstorming more difficult. In this case, the facilitator can assign 10 points to each team member for each possible cause, and let them assign the rating (from 1 to 10, 10 being most likely cause) to each cause. After everyone on the team has rated the causes, the facilitator totals each of the causes and ranks them based on their ratings. From the list, the top four to six causes are identified as major causes and connected as bones in the diagram.

The diagram looks a little like the skeleton of a fish, hence the name Fishbone. After the major causes of the problem are identified, each one of them is discussed in further detail with the team to find out the secondary causes (Whys’). If needed, the secondary causes are further discussed to obtain the next level of possible causes. Each of the major causes is laid as a fishbone in the diagram and the secondary causes as "bonelets," as shown in Fig. 1.

**Rule# 5:** Do not afraid to make the Fishbone diagram complicated. Remember, you have a group of critics which are to screen the generated causes. However, if a correlation is not in the diagram, the critics will not add it.
The diagram now has a comprehensive list of possible causes for the problem, though the list may not be exhaustive or complete. However, the team has enough information to begin discussing the individual causes and to analyze their relevance to the problem. The team can use analytical, statistical, and graphical tools to assist in evaluating each of the causes. The Pareto principle (explained in the next module) is also used to find the elements that cause major problems and to list them as major causes in the Fishbone diagram. Cutting tool performance metrics that are obtained during application support can also be used here for further assistance.

**Evaluate, decide, and take action**

It may be very difficult to come up with consensus on the team of generators for one possible root cause. What should be avoided at any price is democracy, i.e. when a decision is made using the majority because it does not work in engineering and science.

**Rule # 6: Democracy (majority rule) does not work in science and engineering.** If it would than we would continue to believe that the earth is as flat as a pancake, and if you don't believe it you are just another dirty atheist. Pope John Paul II reversed the Roman Catholic condemnation of Galileo only in 1992 – it took so long for people to get read of this old believe. Everyone has heard the story about Nicolas Copernicus (1473-1543) whose treatise “On the revolutions of the heavenly spheres” (published 1543) argued that the earth revolves around the sun. This thesis was directly countered current scientific belief, which held that the earth was the center of the universe and everything revolved around the earth. In both these cases if majority ruled then....you know.

Moreover, democracy if often a prime reason why the error was never fixed: if an error becomes extremely widespread, and hundreds of thousands of people begin making the same mistake, then the error will become invisible. Those people will refuse to even acknowledge the error as being an error. After all, so many people cannot be wrong! Oh yeah? The majority rules? Not where the real world is concerned! It doesn't matter how many people make a factual error: the error remains just as wrong. However, any experts who object, and who try to fix the massive error, they will perhaps be seen as science (engineering)-nitpickers living in ivory towers. The ones who have the ambition to point out the error are easily ignored because they are so few. Therefore, the groups of critics should evaluate all opinions and give really good thoughts to those spelled in a very quiet voice.

The final decision on the root cause is to be made by the critics which evaluate all causes using critical thinking, experience, engineering and scientific principles. Also, the major causes can be ranked in order with the most likely cause at the top of the list. In difficult cases, an expert (experts) in the field should be consulted to make the proper selection. In any case, the feed back in provided to the generators for the training purpose.

The analysis can be taken to deeper level by using Simple Correlation Analysis (See Appendix 1), Regression analysis and more sophisticated statistical tools (see Module 3) to qualify the correlation, and Design of Experiments (see Module 4) to clarify causation. Although both techniques are of a great help in the analysis of possible correlations and “cause- effect” ‘strength’ because they QUNTIFY possible correlations and thus reduce many uncertainties, their deployment requires:

- High qualification of the group members responsible for the evaluation of the completer Fishbone diagram. This qualification implies understanding of basic
statistical principles, experience in evaluation of the experimentally obtained data, calculation of statistical parameters followed by their interpretation and so on.

- Time and funding to collect the field data and to carry out experiments to establish the required correlations.

However, one should realize clearly that the deployment of these methods might very well save the time and money particularly when the root cause is hidden. The final decision is to be made by the six sigma leader who should make a weighted decision remembering that it is not very wise to use a stick if one tries to stop a tank while a submachine gun is of a little help if one wants to kill a fly.

**Rule # 7: The methods used in the establishing the root cause(s) should correspond to the objective of the analysis and the possible impact of its results on the CT manufacturer’s future.**

After the evaluation process is complete, the action plan has to be decided by the six sigma leader. If one possible root cause is identified, then the action plan has to be derived to rectify it.

In some complicated cases, however, it is difficult to identify a single root cause; there may be a few possible root causes with very complicated correlations. In this case, the action plan has to be drawn for each of the possible root causes.

After the action plan is completed, the six sigma leader (provided that he/she is a top company executive) assignees a team to work on the plan and to rectify the problem permanently. If there are a few possible root causes, all the action plans are to be executed, and the most likely root cause is identified and fixed.

**Appendix 1**

**Simple Correlation Analysis**

**Concept of correlation coefficient**

As the Fishbone diagram is developed, one always wonder how to ‘formalize’ the strength of the “cause-effect” relationship. One of the possible ways is to examine the degree of correlation between variables. A statistical measurement of correlation can be calculated using the least squares method to quantify the strength of the relationship between two variables. The output of that calculation is the Correlation Coefficient, or \( r \), which ranges between -1 and 1.

When correlation coefficient is zero then there is no LINEAR correlation between a cause (hereafter is designated as \( x \)) and the effect (hereafter is designed as \( y \)). If one represents the correlation in this case graphically then he/she obtains a picture similar to that shown in Fig.2. Such a representation of data is known as a scatter diagram.
A value of 1 indicates perfect positive correlation - as one variable increases, the second increases in a linear fashion (Figure 3a). Likewise, a value of -1 indicates perfect negative correlation - as one variable increases, the second decreases (figure 3b). Thus the higher the correlation coefficient the stronger LINEAR correlation between cause and effect.

Figure 2. Close to zero correlation between the case (Variable 1) and effect Variable 2).

Figure 3. Close to 1 positive (a) and negative (b) correlations between the case (Variable 1) and effect Variable 2).
Calculation of the correlation coefficient

The first step in calculation of the correlation coefficient is to collect experimental data on the cause \((x)\) and effect \((y)\). This is to collect some experimental data and represent it as a table:

<table>
<thead>
<tr>
<th>(n)</th>
<th>(x)</th>
<th>(y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>(x_1)</td>
<td>(y_1)</td>
</tr>
<tr>
<td>2</td>
<td>(x_2)</td>
<td>(y_2)</td>
</tr>
<tr>
<td>3</td>
<td>(x_3)</td>
<td>(y_3)</td>
</tr>
<tr>
<td>(N)</td>
<td>(x_N)</td>
<td>(y_N)</td>
</tr>
</tbody>
</table>

The second step is to calculate the averages (\(\overline{x}\) and \(\overline{y}\)) of the \(x\) and \(y\) values as

\[
\overline{x} = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{1}{N} \sum_{i=1}^{N} x_i
\]

\[
\overline{y} = \frac{y_1 + y_2 + \ldots + y_N}{N} = \frac{1}{N} \sum_{i=1}^{N} y_i
\]

The third step is to calculate the standard deviations (\(\sigma_x\) and \(\sigma_y\)) of the \(x\) and \(y\) data sets as

\[
\sigma_x = \sqrt{\frac{1}{N} \left[ (x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \ldots + (x_N - \overline{x})^2 \right]}
\]

\[
\sigma_y = \sqrt{\frac{1}{N} \left[ (y_1 - \overline{y})^2 + (y_2 - \overline{y})^2 + \ldots + (y_N - \overline{y})^2 \right]}
\]

and to calculate the covariance between the two data sets as

\[
\sigma_{xy} = \frac{1}{N} \left[ (x_1 - \overline{x})(y_1 - \overline{y}) + (x_2 - \overline{x})(y_2 - \overline{y}) + \ldots + (x_N - \overline{x})(y_N - \overline{y}) \right]
\]

Finally, the correlation coefficient is then defined as

\[
r = \frac{\sigma_{xy}}{\sigma_x \sigma_y}
\]

**EXAMPLE 1:**

Calculate the correlation coefficient for the following \(x\) and \(y\) data sets collected to reveal a possible correlation between the cutting edge radius \((x)\) and tool life \((y)\):
Fishbone analysis
Viktor P. Astakhov

Calculate the averages ($\bar{x}$ and $\bar{y}$) of the x and y values as

$$\bar{x} = \frac{x_1 + x_2 + \ldots + x_N}{N} = \frac{0.005 + 0.010 + 0.015 + 0.020 + 0.025}{5} = 0.015\text{mm}$$

$$\bar{y} = \frac{y_1 + y_2 + \ldots + y_N}{N} = \frac{30 + 45 + 42 + 29 + 18}{5} = 32.8\text{min}$$

Calculate the standard deviations ($\sigma_x$ and $\sigma_y$) as

$$\sigma_x = \sqrt{\frac{1}{N} \left[ (x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \ldots + (x_N - \bar{x})^2 \right]} = \sqrt{\frac{1}{5} \left[ (0.005 - 0.015)^2 + (0.010 - 0.015)^2 + (0.015 - 0.015)^2 + (0.020 - 0.015)^2 + (0.025 - 0.015)^2 \right]} = 0.00707$$

$$\sigma_y = \sqrt{\frac{1}{N} \left[ (y_1 - \bar{y})^2 + (y_2 - \bar{y})^2 + \ldots + (y_N - \bar{y})^2 \right]} = \sqrt{\frac{1}{5} \left[ (30 - 32.8)^2 + (45 - 32.8)^2 + (42 - 32.8)^2 + (29 - 32.8)^2 + (18 - 32.8)^2 \right]} = 9.745$$

and the covariance as

$$\sigma_{xy} = \frac{1}{N} \left[ (x_1 - \bar{x})(y_1 - \bar{y}) + (x_2 - \bar{x})(y_2 - \bar{y}) + \ldots + (x_N - \bar{x})(y_N - \bar{y}) \right] = \frac{1}{5} \left[ (0.005 - 0.015)(30 - 21.8) + (0.010 - 0.015)(45 - 32.8) + (0.015 - 0.015)(0.015 - 0.015) + \ldots \right] = 0.051$$

Finally, the correlation coefficient calculates as
Therefore, one may conclude that the cutting edge radius and tool life have a very strong correlation.

IMPORTANT CONCLUSION REMARKS

1. Correlation measures the strength of a linear relationship between two variables. It's that never-mentioned, often-ignored, qualifier that can trip you up. It's always a good idea to read the fine print, and statistics has a lot of fine print. Knowing that two variables are independent will tell you that their correlation coefficient is zero. But knowing the correlation coefficient to be zero does NOT mean the two variables are independent or otherwise unrelated. It only means there is no linear relationship.

2. Correlation is not Causation - two variables can be very strongly correlated, but both can be caused by a third variable. For example, consider two variables: A) how much my grass grows per week, and B) the average depth of the local reservoir. Both variables could be highly correlated because both are dependent upon a third variable - how much it rains.